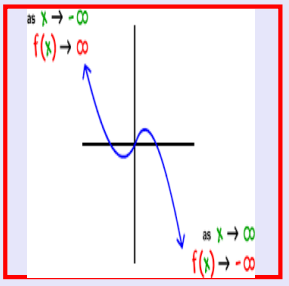
Math 245

Spring 2022

Lecture 38



$P(x) = 3x^4 - 14x^2 - 5$

1) At most 4 Zeros

2) List of all possible Rational Zeros $\frac{\pm 1, \pm 5}{\pm 1, \pm 3} \Rightarrow \pm 1, \pm 5, \pm\frac{1}{3}, \pm\frac{5}{3}$

3) $P(x) = 3x^4 - 14x^2 - 5$ (1 sign change) $\Rightarrow$ 1 pos. Zero

$P(-x) = 3x^4 - 14x^2 - 5$ (1 sign change) $\Rightarrow$ 1 Neg. Zero

+	−	Comp.
1	1	2

4) Is 4 an upper bound?

4	3	0	−14	0	−5
		12	48	136	+
	3	12	34	+	+

Yes

5) Is −3 a lower bound?

−3	3	0	−14	0	−5
		−9	27	−39	117
	3	−9	13	−39	+

Yes

6) Factor $P(x)$

$P(x) = 3x^4 - 14x^2 - 5$

$= (3x^2 + 1)(x^2 - 5)$

7) Find all Zeros

$x^2 - 5 = 0$

$x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

$3x^2 + 1 = 0 \quad 3x^2 = -1$

$x^2 = -\frac{1}{3} \quad x = \pm\sqrt{-\frac{1}{3}}$

$x = \pm\frac{1}{\sqrt{3}}i \quad x = \pm\frac{\sqrt{3}}{3}i$

$P(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$

1) At most 4 Zeros

2) $P(x) = 3x^4 - 4x^3 - 22x^2 + 15x + 18$

1 2

2 or 0 pos. Zeros

$P(-x) = 3x^4 + 4x^3 - 22x^2 - 15x + 18$

1 2

2 or 0 Neg. Zeros

3)

+	−	Comp.
⇒ 2	2	0
2	0	2
0	2	2
0	0	4

4) List of all possible rational Zeros

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$\pm 1, \pm 3$

$\Rightarrow \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{3}, \pm \frac{2}{3}$

4) Is 3 an upper bound?

3 | 3 −4 −22 15 18

9 15 −21 −18

3 5 −7 −6 0

3 is not an upper bound but it is a Zero.

5) Is 3 a repeated Zero?

3 | 3 5 −7 −6 NO

9 42 +

3 14 35 +

6) Is −2 a lower bound?

−2 | 3 −4 −22 15 18

−6 20 4 −38 NO

3 −10 −2 19 20

6) Is $-\frac{2}{3}$ a Zero?

$-\frac{2}{3} \cdot 3 = -2$

$-\frac{2}{3} \cdot (9) = 6$

$-\frac{2}{3}$ | 3 5 −7 −6

−2 −2 6

3 3 −9 0 Yes

7) Factor $P(x)$

$$P(x) = (x-3)\left(x-\frac{-2}{3}\right)(3x^2+3x-9)$$

Zeros

$$P(x) = (x-3)\left(x+\frac{2}{3}\right)\cdot 3(x^2+x-3)$$

$$P(x) = (x-3)(3x+2)(x^2+x-3)$$

8) Find all remaining Zeros

$x^2+x-3=0$

$a=1 \quad b=1 \quad c=-3$

$b^2-4ac = 1^2-4(1)(-3) = 13$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{13}}{2}$$

$$\frac{-1+\sqrt{13}}{2}, \frac{-1-\sqrt{13}}{2}$$

$P(x) = -2x^5+5x^4+34x^3-30x^2-84x+45$

At most 5 Zeros

$P(x) = -2x^5+5x^4+34x^3-30x^2-84x+45$ (1, 2, 3)

3 or 1 Pos. Zeros

$P(-x) = 2x^5+5x^4-34x^3-30x^2+84x+45$ (1, 2)

2 or 0 Neg. Zeros

+	−	Complex
3	2	0
3	0	2
1	2	2
1	0	4

Is 5 a Zero? Yes

5	-2	5	34	-30	-84	45
		-10	-25	45	75	-45
	-2	-5	9	15	-9	0

Is $\frac{1}{2}$ a Zero? Yes

$\frac{1}{2}$	-2	-5	9	15	-9
		-1	-3	3	9
	-2	-6	6	18	0

Is -3 a Zero? Yes

-3	-2	-6	6	18
		6	0	-18
	-2	0	6	0

Factor $P(x) = (x-5)\left(x-\frac{1}{2}\right)(x-(-3))(-2x^2+6)$

$= (x-5)\left(x-\frac{1}{2}\right)(x+3)\cdot 2(-x^2+3)$

$= (x-5)(2x-1)(x+3)(-x^2+3)$

Remaining Zeros $-x^2+3=0$

$-x^2 = -3$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

Pos. Zeros: $5, \frac{1}{2}, \sqrt{3}$ $\Leftarrow$ 3 Pos. Zeros

Neg. Zeros: $-3, -\sqrt{3}$ $\Leftarrow$ 2 Neg. Zeros